Lecture 5 (Nov 23)

Reminder : HW1 due today! (5:30 PM)
office closed.

"Rules for next HW":

- Due right before class starts.
- Get 50% off if you submit late.

Frequency :

You may remember that we can implement basic modulation by changing the instantaneous amplitude, frequency, and/or phase of a sinusoidal signal.

= modulating the said component(s) of the sinusoidal signal with useful information.

$$A \cos(2\pi f t + \phi) \qquad (\star)$$

$A \to$ AM, ASK

$f \to$ FM, FSK

$\phi \to$ PM, PSK

The tricky part is how to modulate the frequency.

Suppose you want to modify the frequency such that the frequency at time t is governed by the expression

$$f(t) = t^2.$$

Can we simply replace the "f" in $(\star)$ above with t^2?

$$A \cos(2\pi (t^2) t + \phi)$$ ← will this gives the right frequency ??

At time $t=2$, we should have $f(2) = 2^2 = 4$ Hz.

At time "near" $t=2$, it looks from the formula above like we have $\cos(2\pi\, 2^2 t + \phi) = \cos(2\pi\, 4t + \phi)$.

Unfortunately, if you plot this function, you will see that

Unfortunately, if you plot this function, you will see that the graph oscillate much faster than 4 Hz.

So the answer to the above question is ... no!

We can not simply "plug-in" the time-dependent freq. formula into (☆).

Here is how to define "instantaneous frequency"

Definition: Given a signal

$$A\cos(\phi(t)),$$

its instantaneous frequency is given by

$$f(t) = \frac{1}{2\pi}\frac{d}{dt}\phi(t).$$

Ex. For $A\cos(2\pi(t^2)t + \phi)$,

$$f(2) = \frac{d}{dt}(t^3)\Big|_{t=2} = 3t^2\Big|_{t=2} = 3\times 4 = 12 \text{ Hz}$$

This indicates that your signal freq. triples the desired freq.

In general, the instantaneous frequency of a signal of the form

$$A\cos(2\pi\, g(t)\, t + \phi)$$

is given by

$$\frac{d}{dt}g(t)\,t = g(t) + \underbrace{t\,g'(t)}$$

↳ this is the part that prevent direct substitution

Ex. If we want $f(t) = t^2$,

we need

$$\phi(t) = 2\pi\int_0^t f(\tau)\,d\tau + C$$

$$= 2\pi\int_0^t \tau^2\,d\tau + C = 2\pi\frac{t^3}{3} + C$$

$$\Rightarrow A\cos\left(2\pi\frac{t^3}{3} + \phi\right)$$

In general, the final answer is

$$A\cos\left(2\pi\int_0^t f(\tau)\,d\tau + \phi\right).$$

Ex. The instantaneous freq. of a signal of the form

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$$\cos\big(\underbrace{2\pi f\left(t - \tfrac{r(t)}{c}\right) + \phi}_{\theta(t)}\big)$$

is given by

$$\frac{1}{2\pi}\frac{d}{dt}\theta(t) = f - \frac{f}{c}\frac{d}{dt}r(t) = \underbrace{f - \frac{1}{\lambda}\frac{d}{dt}r(t)}$$

This is the formula that I used on the slide for doppler shift with angle.

Intuition:

For a signal of the form $A\cos(\phi(t))$, we know, from calculus, that around a specific value of t, say $t = t_0$, we can approximate the function $\phi(t)$ by

$$\phi(t) \approx \phi(t_0) + \phi'(t_0)(t - t_0).$$

$$= \phi'(t_0)\,t + \big(\underbrace{\phi(t_0) - \phi'(t_0)t_0}_{\theta \leftarrow \text{constant}}\big)$$

Hence, near time $t = t_0$, the signal behavior is similar to

$$A\cos\big(\phi'(t_0)\,t + \theta\big).$$

constant

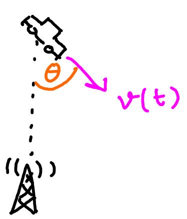
Therefore, near $t = t_0$, the signal can be approximated by a sinusoidal function with frequency $1/2\pi\,\phi'(t_0)$ and phase θ.

This agrees with our formula for instantaneous freq. above.

Lecture 6 (Nov 26)

Review

- Doppler shift (frequency change)



at time $t = 0$

$$f_{new}(0) = f_{original} + \frac{1}{\lambda}\cos\theta\, v(0)$$

- Today: easier derivation via approximation. ✓

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- Instantaneous frequency ✓